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Higgs Boson Decay into Two Gluons and a Z Boson

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Abstract Results for the one-loop calculation of the decay width $\Gamma(H \to ggZ)$ in the standard model with Higgs boson masses in the range 115 GeV $< m_H < 2m_W$ are presented. We find that among all the helicity amplitudes contributing to the width only those for which the gluons have the same polarization and the Z is longitudinally polarized contribute in any significant way. The calculation includes all contributions from the second and third generations, and kinematic cuts to enhance the $H \to ggZ$ signal. Compared to the width of $H \to gg$, we find $\Gamma(H \to ggZ)/\Gamma(H \to gg) \lesssim 10^{-4}$.

Keywords Higgs decay · Triangle anomaly

1 Introduction

The lowest order contributions to the two-body decay $H \to gg$ and the three-body decay $H \to ggZ$ occur at the one-loop level in the standard model. Although the decay width $\Gamma(H \to gg)$ exceeds that of the process $H \to ggZ$ by several orders of magnitude, it is nevertheless possible to identify the $H \to ggZ$ decay by imposing cuts that exclude back-to-back gluon decays and looking for a Z recoiling against jets.

Before the discovery of the top quark, the decay width $\Gamma(H \to ggZ)$ was calculated in [1] for selected values of the top quark mass assuming that the gluons and the Z boson were unpolarized and with no kinematic cuts imposed on the decay products. Here, we extend the calculation of [1] by calculating the decay width as function of the polarizations of the gluons and the Z boson and imposing cuts on the decay products. We find that, effectively, the only possible helicity combinations are the ones where the gluons have the same helicity and the Z boson is longitudinally polarized. We also calculate the distributions of the decay

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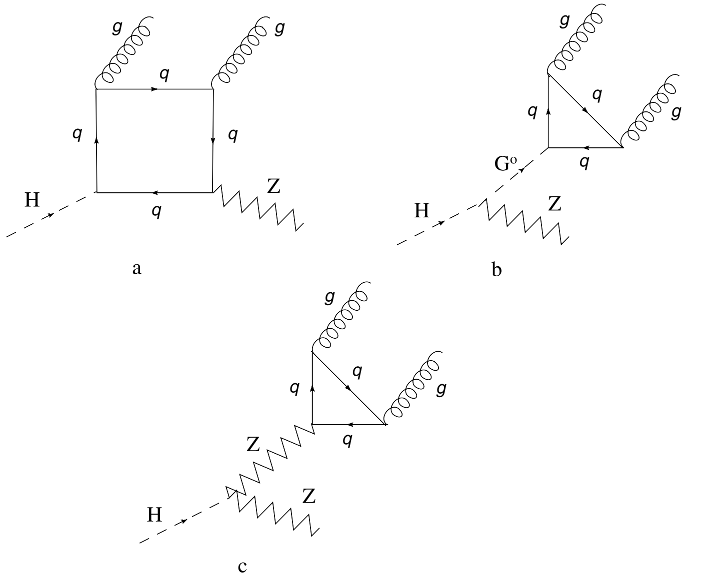

Fig. 1 Sample of Feynman diagrams for the decay process $H \to ggZ$. For the quark q in the loops, we include quarks of the second and third generations

width with respect to the invariant mass of the gluon pair and the energy of the Z boson. In the limit that the top quark is heavy and all other quarks are massless, we recover the approximate result for the decay width given in [1].

In the following section, we present an outline of our calculations of the decay width (with its dependence on the helicities of the gluons and the Z boson), the gluon pair invariant mass distribution, and the Z boson's energy distribution. Finally, we discuss the role of the axial-vector anomaly in the amplitude and its cancellation and end with some conclusions.

2 Calculations and Results

The strategy of our calculations is similar to that of [2], where we used the FeynArts package ([3]; for decay processes, see [4]) and a generalized non-linear gauge fixing condition ([5]; see also [6]). The parameters contained in the gauge fixing terms of this gauge condition do not affect observables and often are chosen to eliminate certain vertices and thereby minimize the number of Feynman diagrams to be evaluated. However, in the decay $H \to ggZ$, each diagram is independent of these parameters and, therefore, the total number of diagrams is fixed. Some representative Feynman diagrams [7, 8] encountered are shown in Fig. 1. In Figs. 1(a) and 1(b), the couplings of the Higgs boson and the neutral Goldstone boson G^0 to the quark are proportional to the quark mass. Because of this, only the contributions of quarks from the second and third generations are included.

For a given quark, the amplitude $\mathcal{A}_T(m_q)$, corresponding to the triangle diagram of Fig. 1(c), is proportional to the axial-vector coupling constant $g_A^q = gT_3^q$, where T_3^q is the third component of the quark's weak isospin. As far as the dependence on the quark mass m_q is concerned, the amplitude can be decomposed as $\mathcal{A}_T(m_q) = \mathcal{A}_T(0) + (\mathcal{A}_T(m_q) - \mathcal{A}_T(0)) \equiv \mathcal{A}_T(0) + \mathcal{A}_m$. The $\mathcal{A}_T(0)$ term is nonzero and represents the anomalous contribution (for a discussion of the anomalies, see [9]; for a calculation of an anomalous triangle diagram, see [10]). However, the inclusion of both quarks of a given generation will cancel this anomalous contribution since $\sum_q g_A^q = 0$. Furthermore, if both quarks of a particular generation had the same mass, they would have the same $\mathcal{A}_m$ (apart from an overall factor of g_A^q) and the total contribution of that generation would vanish. Here, we include only the quarks of the second and third generations in the evaluation of the diagram in Fig. 1(c).

The decay process $H \to ggZ$ is small compared to the decay $H \to gg$. To facilitate the discrimination of gluons from these two decay modes and account for some of the possible experimental limitations, we impose the following cuts (as discussed in [2]) on the 3-momenta, invariant mass, and opening angles of the decay products of the process $H \to ggZ$:

$$|\vec{p}_g|, |\vec{p}_Z| > |\vec{p}|_{\text{cut}}, \tag{1}$$

$$m_{gg} > m_{\text{cut}}, \tag{2}$$

$$\pi - \theta_{\text{cut}} > \theta_{gZ}, \quad \theta_{gg} > \theta_{\text{cut}}. \tag{3}$$

For our numerical calculations, we choose $|\vec{p}\,|_{\text{cut}} = 5$ GeV, $m_{\text{cut}} = 12$ GeV, and $\theta_{\text{cut}} = \pi/18$. These cuts provide minimum opening angles between the gluons and Z boson, exclude contributions of the back-to-back and low energy gluons, and also improve the numerical stability of the calculations. The cuts help discriminate the non-back-to-back gluons of the decay $H \to ggZ$ from the back-to-back gluons of the decay $H \to gg$. In principle, all the gluons of these two decay modes can be identified.

We calculated the decay width and its distributions with respect to the invariant mass of the gluon pair and the energy of the Z boson, and checked the gauge invariance of the results with respect to the gluons. (The diagrams of Figs. 1(a), 1(b), and 1(c) are separately gauge invariant.) Due to Bose symmetry and CP invariance, not all helicity amplitudes are independent. In fact, our numerical calculation of the decay widths $\Gamma_{\lambda\lambda'\lambda_Z}(H \to ggZ)$, where λ and λ' are the helicities of the gluons and λ_Z is the helicity of the Z boson, showed that effectively only the helicity combinations Γ_{--0} and Γ_{++0} are nonzero and satisfy the relation

$$\Gamma_{--0} = \Gamma_{++0}. \tag{4}$$

We will elaborate on this point in the conclusions.

The decay width $\Gamma(H \to ggZ)$ as function of the Higgs boson mass m_H is shown in Fig. 2. For comparison, in this figure we also include the decay width $\Gamma(H \to gg)$, which was calculated using the HDECAY package [11] and the width $\Gamma(H \to ggZ)$ as calculated in [1] for $m_t \to \infty$. The effect of the cuts imposed on the decay products can be seen in difference between the two $\Gamma(H \to ggZ)$ curves. It is clear from this figure that the ratio of the decay widths is small, $\Gamma(H \to ggZ)/\Gamma(H \to gg) \lesssim 10^{-4}$. The smallness of this ratio is due to the combination of the imposed cuts on the decay products, the suppression from the three-body phase space, the higher order in the coupling constant, and the inclusion of the anomalous triangle diagram of Fig. 1(c) in the decay $H \to ggZ$.

In Fig. 3, the invariant mass distribution $d\Gamma(H \to ggZ)/dm_{gg}$ as function of the gluon pair invariant mass m_{gg} is shown, and Fig. 4 shows the energy distribution $d\Gamma(H \to ggZ)/dE_Z$ as function of the Z boson energy E_Z. These figures show that, in the Higgs rest frame, the m_{gg} is always relatively small and the Z is nearly at rest. This situation is rather different from that of a ggZ state produced in H-Z associated production followed by the decay $H \to gg$.

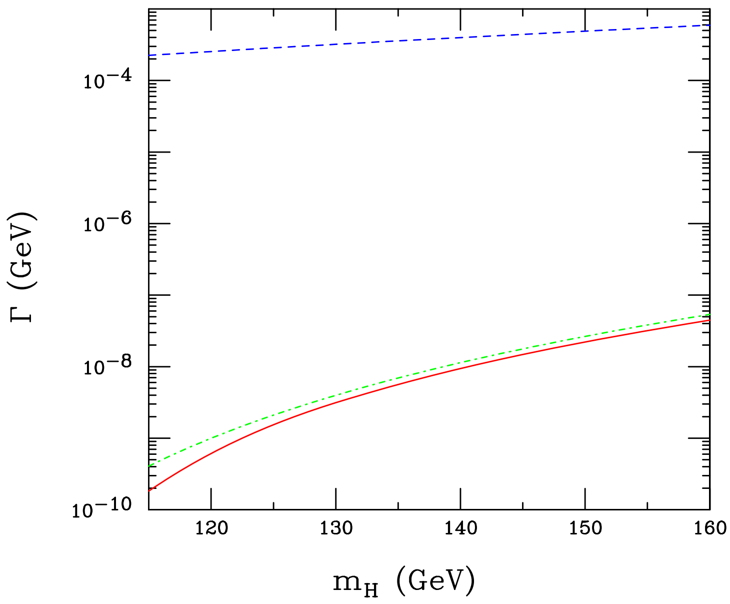

Fig. 2 The decay widths as function of m_H for two decay modes of the Higgs boson are shown. The *solid line* is our result $\Gamma(H \to ggZ)$, the *dashed line* is the result for $\Gamma(H \to gg)$ using [11] and the *dot-dashed line* is the result for $\Gamma(H \to ggZ)$ with $m_t \to \infty$ from [1]

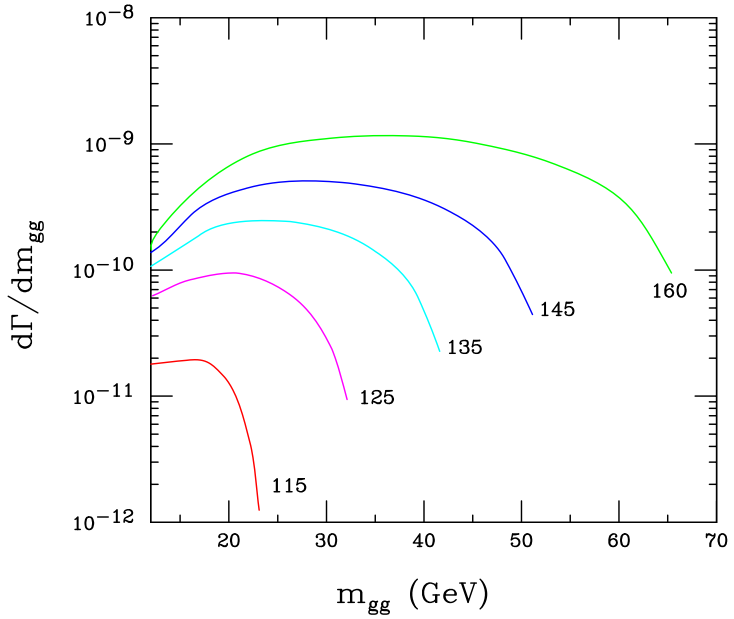

Fig. 3 The invariant mass distributions $d\Gamma(H \to ggZ)/dm_{gg}$ as function of m_{gg}, the invariant mass of the gluon pair, for Higgs boson masses of m_H = 115, 125, 135, 145, and 160 GeV are shown

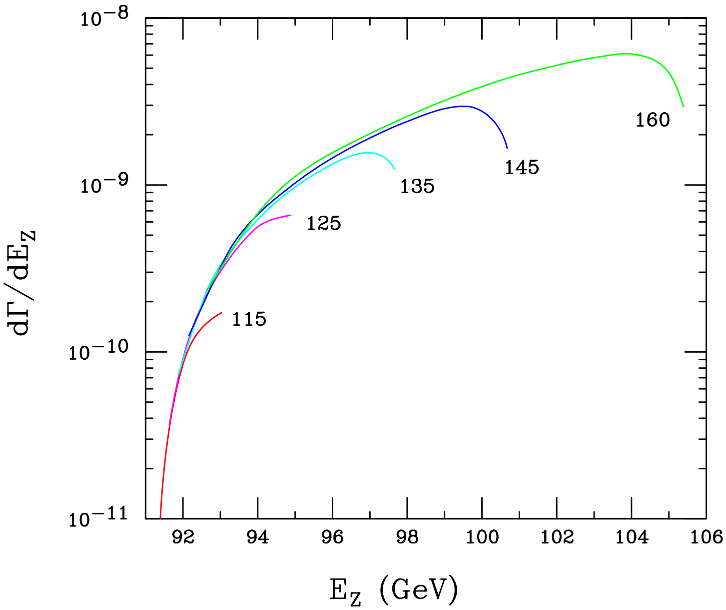

Fig. 4 The energy distributions $d\Gamma(H \to ggZ)/dE_Z$, as function of the Z gauge boson energy E_Z, for the Higgs boson masses of Fig. 3 are shown

3 Conclusions

We find that the decay width for the process $H \to ggZ$ is small compared to that of $H \to gg$, $\Gamma_{H\to gZ}/\Gamma_{H\to gg} \lesssim 10^{-4}$. A feature of this decay mode that is not shared with the decay mode $H \to gg$ is the presence of an anomalous vertex in the diagram of Fig. 1(c). If a sample of $H \to ggZ$ decays could be obtained, it would be interesting to check the validity of the anomaly cancellation.

Our explicit calculations of the helicity amplitudes for the decay $H \to ggZ$ show that the only helicity combinations of any appreciable size are those for which the gluons have the same helicity and the Z boson is longitudinally polarized. In the limit that the light quarks are massless and the top quark mass is large, this property can be traced to the structure of the triangle diagram [12].

Neglecting the Z width, the triangle amplitude $\mathcal{A}_T(m_q)$ obtained from Figs. 1(b) and 1(c) is

$$\mathcal{A}_T(m_q) = -\frac{g^2 T_3^q\, \delta_{ab}}{m_Z \cos^2\theta_W} \frac{\alpha_S}{\pi} \frac{q\cdot\xi^*}{q^2}\left[\frac{1}{2} + \frac{m_q^2}{q^2} F\left(\frac{q^2}{m_q^2}\right)\right]\varepsilon_{\lambda\rho\mu\nu} k^\mu k'^\nu \varepsilon^{\lambda *}\varepsilon'^{\rho *}, \tag{5}$$

where $\xi(p')$ is the polarization vector of the Z, $\varepsilon(k)$ and $\varepsilon'(k')$ are the gluon polarization vectors, δ_{ab} is the color factor, $q = k + k'$, the $\frac{1}{2}$ is the anomalous contribution and

$$F\left(\frac{q^2}{m_q^2}\right) = \int_0^1 \frac{dx}{x} \ln\left[1 - \frac{q^2}{m_q^2} x(1-x) - i\varepsilon\right]. \tag{6}$$

Summing over the quarks of a given generation eliminates the anomaly and leaves

$$\mathcal{A}_T = -\frac{g^2\delta_{ab}}{m_Z\cos^2\theta_W}\frac{\alpha_S}{2\pi}\frac{q\cdot\xi^*}{q^2}\varepsilon_{\lambda\rho\mu\nu}k^\mu k'^\nu\varepsilon^{\lambda *}\varepsilon'^{\rho *}\left[\frac{m_1^2}{q^2}F\left(\frac{q^2}{m_1^2}\right)-\frac{m_2^2}{q^2}F\left(\frac{q^2}{m_2^2}\right)\right]. \quad (7)$$

In the approximation that the light quarks u, d, s, c, b have zero mass and the top mass is large compared to the Higgs mass, we have $\lim_{m^2\to 0} m^2F(q^2/m^2)\to 0$ and

$$\lim_{m^2\gg q^2}\frac{m^2}{q^2}F\left(\frac{q^2}{m^2}\right)\to -\frac{1}{2}-\frac{1}{24}\frac{q^2}{m^2}. \quad (8)$$

For this case, the leading contribution to $\mathcal{A}_T$ can be attributed to the top quark and is

$$\mathcal{A}_T = \frac{g^2\delta_{ab}}{m_Z\cos^2\theta_W}\frac{\alpha_S}{4\pi}\frac{p\cdot\xi^*}{q^2}\varepsilon_{\lambda\rho\mu\nu}k^\mu k'^\nu\varepsilon^{\lambda *}\varepsilon'^{\rho *}, \quad (9)$$

where $p = p' + k + k'$ is the Higgs momentum and we have used $q = p - p'$ together with $p'\cdot\xi = 0$. Note that m_t does not explicitly appear in (9). To obtain the helicity dependence of $\mathcal{A}_T$, we can work in the rest frame of the Higgs boson. Then, the only Z polarization that contributes is $\lambda_Z = 0$, which gives $p\cdot\xi^* = m_H|\vec{p}'|/m_Z$. In this frame, the three-momenta of the Z and the two gluons all lie in the same plane with $\vec{p}' = -\vec{k}-\vec{k}'$, and it is straightforward to show that $\varepsilon_{\lambda\rho\mu\nu}k^\mu k'^\nu\varepsilon^{\lambda *}\varepsilon'^{\rho *}\to -i(\lambda+\lambda')q^2/4$. The triangle helicity amplitudes $\mathcal{A}_{\lambda\lambda'}$ are then given by

$$\mathcal{A}_{\lambda\lambda'} = -i\frac{G_F\delta_{ab}}{2\sqrt{2}}\frac{\alpha_S}{\pi}m_H(\lambda+\lambda')|\vec{k}+\vec{k}'|, \quad (10)$$

and it is clear that the only non-vanishing helicity configurations are those for which the Z boson is longitudinally polarized and the gluons have the same helicity. When (10) is used to calculate $\Gamma(H\to Zgg)$, the result, setting $r = m_Z^2/m_H^2$, is

$$\Gamma(H\to Zgg) = \frac{G_F^2 m_H^5}{1024\pi^3}\left(\frac{\alpha_S}{\pi}\right)^2[1-8r+8r^3-r^4-12\ln(r)], \quad (11)$$

in agreement with [1]. For completeness, note that the non-leading term in (8) contributes,

$$\delta\mathcal{A}_T = \frac{g^2\delta_{ab}}{m_Z\cos^2\theta_W}\frac{\alpha_S}{48\pi}\frac{p\cdot\xi^*}{m_t^2}\varepsilon_{\lambda\rho\mu\nu}k^\mu k'^\nu\varepsilon^{\lambda *}\varepsilon'^{\rho *}, \quad (12)$$

which exactly cancels the leading order $1/m_t^2$ contribution from the box diagram, Fig. 1(a), which is calculated in [12]. As a consequence, the remaining contributions are of order $1/m_t^4$ and therefore highly suppressed.

With enough data, the presence of a longitudinally polarized Z in an event with an opposite side low invariant mass jet pair could be useful in identifying Z's from $H\to ggZ$ decays.

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